Consider the vector $\vec{m} = \langle -2, -4 \rangle$, and the vector $\vec{n}$ with direction angle $\frac{2\pi}{3}$ such that $\|\vec{n}\| = 8$. **SCORE: _____ / 5 PTS**

[a] Find a unit vector perpendicular to $\vec{m}$. (Do **NOT** use decimal approximations.)

$\vec{u} = \langle u_1, u_2 \rangle$

$\vec{m} \cdot \vec{u} = 0 \Longrightarrow -2u_1 - 4u_2 = 0$ eg. $u_1 = -2,\ u_2 = 1$

$\frac{\langle -2, 1 \rangle}{\|\langle -2, 1 \rangle\|} = \frac{1}{\sqrt{5}} \langle -2, 1 \rangle = \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$ ALSO $\left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

[b] Find the direction angle of $\vec{m}$. (Your answer should be in radians, rounded to 2 decimal places.)

$\|\vec{m}\| = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$\sin\theta = \frac{-4}{2\sqrt{5}} = -\frac{2}{\sqrt{5}}$ θ IN Q_3

$\cos\theta = \frac{-2}{2\sqrt{5}} = -\frac{1}{\sqrt{5}}$ $\theta_{REF} \approx 1.11$

$\tan\theta = \frac{-4}{-2} = 2$ $\theta \approx \pi + 1.11 = 4.25$

[c] Write $2\vec{n} - \vec{m}$ as a linear combination of $\vec{i}$ and $\vec{j}$. (Do **NOT** use decimal approximations.)

$2\left(\left(8\cos\frac{2\pi}{3}\right)\vec{i} + \left(8\sin\frac{2\pi}{3}\right)\vec{j}\right) - \left(-2\vec{i} - 4\vec{j}\right)$

$= -8\vec{i} + 8\sqrt{3}\,\vec{j} + 2\vec{i} + 4\vec{j}$

$= -6\vec{i} + (4 + 8\sqrt{3})\vec{j}$

Consider the vectors $\vec{f} = 2\vec{j} - 3\vec{k}$ and $\vec{g} = -\vec{i} - 3\vec{j} + 4\vec{k}$.

SCORE: _____ / 6 PTS

[a] Find the angle between $\vec{f}$ and $\vec{g}$. (Your answer should be in radians, rounded to 2 decimal places.)

$$\cos^{-1} \frac{\vec{f} \cdot \vec{g}}{\|\vec{f}\|\|\vec{g}\|} = \cos^{-1} \frac{-6-12}{\sqrt{13}\sqrt{26}}$$

$$= \cos^{-1} \frac{-18}{13\sqrt{2}}$$

$$\approx 2.94$$

[b] Find a unit vector perpendicular to both $\vec{f}$ and $\vec{g}$. (Do **NOT** use decimal approximations.)

$$\vec{f} \times \vec{g} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -3 \\ -1 & -3 & 4 \end{vmatrix} = 8\vec{i} + 3\vec{j} - 9\vec{i} + 2\vec{k}$$

$$= \langle -1, 3, 2 \rangle$$

$$\frac{\langle -1,3,2 \rangle}{\|\langle -1,3,2 \rangle\|} = \frac{1}{\sqrt{14}} \langle -1,3,2 \rangle = \left\langle -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right\rangle$$

[c] Write $\vec{f}$ as the sum of 2 vectors, one parallel to $\vec{g}$ and one perpendicular to $\vec{g}$. (Do **NOT** use decimal approximations.)

$$\text{PROJ}_{\vec{g}} \vec{f} = \frac{\vec{f} \cdot \vec{g}}{\vec{g} \cdot \vec{g}} \vec{g}$$

$$= \frac{-18}{26} \langle -1, -3, 4 \rangle$$

$$= \left\langle \frac{9}{13}, \frac{27}{13}, \frac{-36}{13} \right\rangle$$

$$\vec{f} - \text{PROJ}_{\vec{g}} \vec{f} = \langle 0, 2, -3 \rangle - \left\langle \frac{9}{13}, \frac{27}{13}, \frac{-36}{13} \right\rangle$$

$$= \left\langle -\frac{9}{13}, -\frac{1}{13}, \frac{-3}{13} \right\rangle$$

$$\vec{f} = \left\langle \frac{9}{13}, \frac{27}{13}, \frac{-36}{13} \right\rangle + \left\langle -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} \right\rangle$$

Let P be the point $(-5,-2,3)$. Let Q be the point $(3,2,-1)$. Let R be the point $(-3,4,-2)$. SCORE: _____ / 19 PTS

Let $\vec{u}$ be the vector with initial point R and terminal point Q.

Let $\vec{w}$ be the vector with initial point P and terminal point R.

[a] In which octant is R ?

$O_{2+4} = O_6$

[b] If you start at point P, move 2 units down, 4 units back, and 6 units to the right, find the co-ordinates of your ending point.

$(-5-4, -2+6, 3-2) = (-9,4,1)$

[c] Write $\vec{u}$ in component form.

$\langle 3--3, 2-4, -1--2 \rangle = \langle 6,-2,1 \rangle$

[d] Write $\vec{w}$ as a linear combination of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

$(-3--5)\vec{i} + (4--2)\vec{j} + (-2-3)\vec{k} = 2\vec{i} + 6\vec{j} - 5\vec{k}$

[e] Find the magnitude of $\vec{w}$. (Do **NOT** use decimal approximations.)

$\sqrt{4+36+25} = \sqrt{65}$

[f] Find a unit vector in the opposite direction as $\vec{w}$. (Do **NOT** use decimal approximations.)

$-\frac{1}{\sqrt{65}}\langle 2,6,-5 \rangle = \left\langle -\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}} \right\rangle$

[g] Find a vector of magnitude 6 in the same direction as $\vec{u}$. (Do **NOT** use decimal approximations.)

$$6\left(\frac{\vec{u}}{\|\vec{u}\|}\right) = 6 \cdot \frac{1}{\sqrt{36+4+1}}\langle 6,-2,1 \rangle$$
$$= \frac{6}{\sqrt{41}}\langle 6,-2,1 \rangle$$
$$= \left\langle \frac{36}{\sqrt{41}}, \frac{-12}{\sqrt{41}}, \frac{6}{\sqrt{41}} \right\rangle$$

[h] If $\|\vec{v}\| = 3$, and the angle between $\vec{u}$ and $\vec{v}$ is 2 radians, find $\vec{u} \cdot \vec{v}$. (Round your answer to 2 decimal places.)

$\|\vec{u}\|\,\|\vec{v}\| \cos 2 = (\sqrt{41})(3)\cos 2 \approx -7.99$

CONTINUED ON OTHER SIDE ➡

[i] If $\|\vec{v}\| = 3$, and the angle between $\vec{u}$ and $\vec{v}$ is 2 radians, find the magnitude of $\vec{u} \times \vec{v}$. (Round your answer to 2 decimal places.)

$$\|\vec{u}\|\|\vec{v}\| \sin 2 = (\sqrt{41})(3) \sin 2 \approx 17.47$$

[j] Find the area of triangle PQR. (Do **NOT** use decimal approximations.)

$$\vec{u} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -2 & 1 \\ 2 & 6 & -5 \end{vmatrix} = \begin{matrix} 10\vec{i} + 2\vec{j} + 36\vec{k} \\ -6\vec{i} + 30\vec{j} + 4\vec{k} \end{matrix}$$

$$= \langle 4, 32, 40 \rangle$$

$$\|\vec{u} \times \vec{w}\| = \|4\langle 1, 8, 10 \rangle\|$$
$$= 4\sqrt{1 + 64 + 100}$$
$$= 4\sqrt{165}$$

$$\text{AREA} = \tfrac{1}{2}(4\sqrt{165}) = 2\sqrt{165}$$

[k] Find the general equation of the plane passing through P, Q and R.

NORMAL VECTOR = $\langle 4, 32, 40 \rangle$ OR $\langle 1, 8, 10 \rangle$

$$x + 8y + 10z = 1(3) + 8(2) + 10(-1)$$
$$x + 8y + 10z = 9$$

[l] Find parametric equations for the line which passes through P and is also parallel to $\vec{u}$.

$$x = -5 + 6t$$
$$y = -2 - 2t$$
$$z = 3 + t$$

[m] Find symmetric equations for the line which passes through Q and is also perpendicular to the plane $-2x - 3y + z = 9$.

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z+1}{-1} \text{ OR } -z-1$$

DIRECTION VECTOR = NORMAL VECTOR
$= \langle -2, -3, 1 \rangle$
OR $\langle 2, 3, -1 \rangle$

[n] Find the equation of the sphere with P and Q as endpoints of a diameter.

$$\text{CENTER} = \text{MIDPOINT} = \left(\frac{-5+3}{2}, \frac{-2+2}{2}, \frac{3-1}{2}\right) = (-1, 0, 1)$$

$$\text{RADIUS} = \sqrt{(3-1)^2 + (2-0)^2 + (-1-1)^2} = \sqrt{16+4+4} = \sqrt{24}$$

$$(x+1)^2 + y^2 + (z-1)^2 = 24$$

[o] Find the equation of $yz-$trace of the sphere with P and Q as endpoints of a diameter.

$$(0+1)^2 + y^2 + (z-1)^2 = 24$$
$$y^2 + (z-1)^2 = 23$$